Reflections on non-linear seismic inversion

by Knut Sørsdal Oslo 2019

Seismic inversion attempts to extract spatially variable physical parameters from measured seismic data. These physical parameters may be representative of the Earth's subsurface media, and have physical and geological meanings, and thus seismic inversion is a quantitative interpretation of seismic measurement. The inversion procedure is generally nonlinear, as the entire inversion engine to solve the inverse problem, at least partly, depends upon the solution. Seismic inversion uses forward modeling to generate synthetic seismic data that will match the observed seismic data. Data fitting is a principal part of the objective function. The distance is measured in a way that we regard reflectivity per depth unit. Regularisation considers the properties of the mapping operator M from the mathematical viewpoint: Whether the numerical instability comes from the singularity, and whether the singular operator can be modified to stabilize the computation.

Thus, forward modeling can be presented on a linear form:

$$Mm=r \qquad (1.1)$$

Where M is a geophysical operator (a matrix), m is the 'model' vector and r is the 'data' vector. Both vectors m and r are defined in the Hilbert space. Also the matrix M is defined in the Hilbert space.

Originally, seismogram is recorded in time-domain and sampled with a total of NT samples. N is number of samples and T is sample interval.

Applying a data-fitting objective function as

$$\phi(m)=\| \vec{r} \text{ -Mm } \|^{2}$$

where r is the observed data vector and $\|r\|^2 = r^T r = (r,r) = \sum_i r_i^2$ is the inner product of a single vector r. The symbol ||.|| represents the L_2 norm of a vector.

The least squares solution using a minimal variation principle, that is, setting $d\phi/dm = 0$ leads to the following linear system.

$$(M^T * M)^T * \vec{r} = M^T \vec{K} \Rightarrow (M^T M)\vec{r} = M^T * \vec{K}$$

$$\Rightarrow \vec{r} = (M^T * M)^{-1} M^T \vec{K} \qquad (1.2)$$

So, we have the observed data r and the 'inverted' data $\vec{r}$ presented by Eq (1.1) and Eq (1.2).

Let us now compare two linear systems presented in Equations 1.1 and 1.2. respectively. Equation 1.1 is a direct problem:

(model m) → [direct mapping M] → data r

Given an Earth model m, defined as a set of Earth parameters, and a mapping operator M, the object is to find a set of data r containing all possible measurements in the data space. Even for this direct problem, m and M may not be unique for a practical problem. For instance, the acoustic, the elastic or the viscoelastic wave all can be used for the problem of generating synthetic reflection seismograms.

The selection is made based on the practical requirements and a priori information. For a correctly defined physical problem, the direct problem is usually well-posed, which means this mathematical model that describes physical phenomena does have a unique solution, and the solution depends continuously on the model. The dependency means that a small variation in the model space M causes a small perturbation Δr in the data space r.

2.Non-linear modeling employing the Riccati equation

The basic idea of this research was first published on Reserachgate.net (Sørsdal (2018)). We follow these lines:

Assume monochromatic plane-waves propagating along a vertical axis. Let P define the stress (pressure) and W the displacement. Density is ρ. Newton's second law gives:

$$\frac{dP}{dz}+\rho\omega^2 W=0 \tag{2.1}$$

Correspondingly, a stress-strain relationship of the following form is assumed (Hook's law):

$$P=\rho\upsilon_r^2 Y\frac{dW}{dz} \tag{2.2}$$

In Eq.(2.2) v_r is the reference velocity which could be taken as the group velocity in case of dispersion. The function Y represents depth and frequency-dependent absorption.

In case of no damping, Y=1 and Eq.(2.2) is simply Hookes law.

Combination of Eqs.(2.1) and (2.2) gives Helmholtz equation (assume constant density)

$$\frac{d^2P}{dz^2}+k^2P=0\ ,k=\frac{\omega}{\upsilon_r\sqrt{Y}} \tag{2.3}$$

In case of a layer with constant velocity and absorption function Y, plane-wave solutions of Eq. (2.3) can be written formally on the simple form (positive z-axis pointing downwards)

$U=a\exp[ikz]$, $D=b\exp[-ikz]$, $k=k_r-k_i$ ($k_r,k_i\geq 0$) (2.4)

With U and D representing respectively upward and downward propagating components. Thus, the total field can be written

$P=U+D$ (2.5)

From eq. (2.1) and by using Eqs.(2.4) and (2.5) it follows that

$$W=-\frac{1}{i\omega\rho v_r\sqrt{Y}}(D-U) \tag{2.6}$$

A depth-varying model can be assumed as the limit of an infinite number of infinitesimal layers. For such a model, the relation in Eq. (2.6) is also assumed to be valid. Differentiation of Eq.(2.6) with respect to z gives (also making use of Eq.(2.1))

$$\frac{dW}{dz} = \frac{1}{\rho v_r^2 Y} P = \frac{1}{\rho v_r^2 Y}(U + D) = -\frac{1}{i\omega\rho v_r \sqrt{Y}}\left(\frac{dD}{dz} - \frac{dU}{dz}\right) - \frac{1}{i\omega}\frac{d}{dz}\left(\frac{1}{\rho v_r \sqrt{Y}}\right)(D - U) \quad (2.7)$$

Consider now the last term on the RHS of Eq.(2.7), a further simplification can be obtained by studying (small to moderate absorption)

$$\frac{d}{dz}\left(\frac{1}{\rho v_r \sqrt{Y}}\right) = -\frac{1}{\rho v_r \sqrt{Y}}\left[\frac{1}{\sqrt{Y}}\frac{d\sqrt{Y}}{dz} + \frac{1}{\rho v_r,}\frac{d(\rho v_r)}{dz}\right] \cong -\frac{2}{\rho v_r \sqrt{Y}} r\,, \quad (2.8)$$

$$r(z) = \frac{i}{2\rho v_r}\frac{d(\rho v_r)}{dz}$$

Where r represents the depth-dependent ‘reflectivity’ per depth unity

Combination of Eqs.(2.1) and (2.6) gives

$$\frac{dP}{dz} = \frac{dU}{dz} + \frac{dD}{dz} = \frac{\omega}{i v_r \sqrt{Y}}(D - U) \quad (2.9)$$

A main result is obtained now by combining Eqs. (2.7)-(2.9)

$$\begin{aligned} \frac{dD}{dz} &= -\frac{i\omega}{v_r \sqrt{Y}} D + r\,(D - U) \\ \frac{dU}{dz} &= \frac{i\omega}{v_r \sqrt{Y}} U - r\,(D - U) \end{aligned} \quad (2.10)$$

Introduce now the ratio K=U/D that is the reflection coefficient, and differentiate it with respect to depth

$$\frac{dK}{dz} = \frac{d(U/D)}{dz} = \frac{1}{D}\frac{dU}{dz} - \frac{U}{D^2}\frac{dD}{dz} \quad (2.11)$$

Finally, by combining Eqs.(2.10) and (2.11) gives the Riccati equation (Gjevik et al, 1976)

$$\frac{dK}{dz} = \frac{2i\omega}{v_r \sqrt{Y}} K - r\,(1 - K^2) \quad (2.12)$$

Since vertically travelling waves are considered, the transformation from depth to two-way traveltime is straightforward

$$\tau = 2\int_0^z \frac{dz}{v_r}, \;\Rightarrow d\tau = \frac{2}{v_r} dz \quad (2.13)$$

Which gives the travel time version of Eq.(2.29)

$$\frac{dK(\omega,\tau)}{d\tau} = \frac{i\omega}{\sqrt{Y(\omega,\tau)}} K(\omega,\tau) - r(\tau)(1 - K^2), r(\tau) = \frac{i}{2\rho v_r}\frac{d(\rho v_r)}{d\tau} \quad (2.14)$$

By noticing that

$\exp(-i\omega\int_0^{\tau} Y(\omega,\tau))^{-1/2}\,d\tau) \equiv \exp\left[-\phi(\omega,\tau)\right]$ is an integrating factor for this Riccati equation, it can be rewritten on the following form:

$$\frac{d}{d\tau}\left[K(\omega,\tau)\exp(-\phi(\omega,\tau)\right] = -r(\tau)(1-K^2)\ \exp(-\phi(\omega,\tau) \qquad (2.15)$$

Where

$$\phi(\omega,\tau) = i\omega\int_0^{\tau}\frac{d\tau}{\sqrt{Y(\omega,\tau)}} \qquad (2.16)$$

Assume now the following boundary condition: K=0 when τ≥T. Integration of Eq. (2.15) now gives the solution

$$-K(\omega,\tau)\exp(-\phi(\omega,\tau)) = -\int_{\tau}^{T} r(\tau)\exp(-\phi(\omega,\tau'))(1-K^2(\omega,\tau'))\,d\tau' \qquad (2.17)$$

Which can be arranged as

$$K(\omega,\tau) = \exp(\phi(\omega,\tau))\int_{\tau}^{T} r(\tau')\exp(-\phi(\omega,\tau'))(1-K^2(\omega,\tau'))\,d\tau' \qquad (2.18)$$

Equation (18) is now the starting point for a modeling algorithm.

Choice of absorption model

Following Horton (1959) we introduce the notation for the absorption function Y

$$Y(\omega,\tau) = A(\omega,\tau) + iB(\omega,\tau) \qquad (2.19)$$

In his paper, Horton gives examples of values of A and B for various absorption models that can be causal or non-causal. Employing Eq (2.3) the wavenumber *k* can now be expressed as (B<(<)A)

$$k = \frac{\omega}{v_r\sqrt{Y}} = \frac{\omega}{v_r\sqrt{A+iB}} = \frac{\omega}{v_r}\left[\frac{1}{\sqrt{A}} - \frac{i}{2}\frac{B}{A\sqrt{A}}\right] \qquad (2.20)$$

Our objective is to find a set of A and B which fulfills the causality criterion. For a more complete discussion about dispersion and attenuation models, the reader is referred to Ursin and Toverud (2002). Sørsdal (1981) was the first to introduce Eq. (2.20) into the Riccati equation. In case of a constant-Q model (Kjartansson, 1979), the wavenumber *k* can be written on the following form

$$k = \frac{\omega}{v(\omega)}\left[1-\frac{i}{2Q}\right] = \frac{\omega}{v_r} + \left[\frac{\omega}{v(\omega)} - \frac{\omega}{v_r}\right] - i\alpha(\omega) = \frac{\omega}{v_r} + \varphi(\omega) - i\alpha(\omega).\ \ \alpha = \frac{\omega}{2Qv(\omega)} \qquad (2.21)$$

Where α is the absorption coefficient and φ is the phase of the ‘absorption filter’. In order to ensure causality, the filter should be minimum phase.

We introduce a Kolsky type of phase-velocity model (Kolsky, 1956)

$$v(\omega) = v_h \left(\frac{\omega}{\omega_h} \right)^{\gamma} \ , \ \gamma = (\pi Q)^{-1} \tag{2.22}$$

Which in combination with Eq.(21) gives

$$k = \frac{\omega}{v_h} \left[1 + \left[\left(\frac{\omega}{\omega_h} \right)^{-\gamma} - 1 \right] - \frac{i}{2Q} \left(\frac{\omega}{\omega_h} \right)^{-\gamma} \right] \tag{2.23}$$

Direct comparison between Eqs. (20) and (23) gives

$$A = \left[\frac{\omega}{\omega_r} \right]^{2\gamma} \quad B = \left[\frac{\omega}{\omega_r} \right]^{2\gamma} \frac{1}{Q} \tag{2.24}$$

In the actual application (previous paper) we followed Wang (2008) and choosed the highest possible (tuning) frequency of the signal band as the reference. With time-sampling $\Delta\tau$=0.004 s this will be the Nyquist frequency 125 Hz. Wang called this model ‘modified Kolsky’ This absorption model is assumed to be causal, at least in an approximate way, since it is closely related to the power law of Strick(1967) which satisfies the Kramers-Krönigs relations.

Non-linear inversion employing the Riccati equation

The inverse problem in Equation (1.2) states that, given a data set r and the mapping operator M, to find a model $\vec{r}$.

Consider now Eq. (2.18) in the limit $\tau \to 0$, which gives the ‘seismogram’

$$K(\omega,0) = \int_0^T r(\tau') \exp(-\phi(\omega,\tau'))(1 - K^2(\omega,\tau'))\, d\tau' \tag{2.25}$$

Introduce ‘reflectivity’ series

$$r(\tau) = \Delta\tau \sum_{i=0}^{NT-1} r_i \, \delta(\tau - i\Delta\tau) \ , T = NT.\Delta\tau \tag{2.26}$$

Combination of Eqs. (25) and (26) gives

$$K(\omega,0) = \sum_{i=0}^{NT-1} r_i \exp(-\phi(\omega, i\Delta\tau))(1 - K^2(\omega, i\Delta\tau))\, \Delta\tau \tag{2.27}$$

Originally, seismogram recorded in timedomain, i.e. k(t,0), and assume sampled with a total of NT-samples. Fourier transform of the data will give the same number of monochromatic seismograms.

Hagos (2016) presented the theory in his thesis and made some computations based on the reflectivity. We have also made some calculations based on the impedance as Gjevik did in his paper. To do this we have to introduce the matrix system Eq.(2.28).

$$\begin{bmatrix} K_{n+1}(\omega_0,0) \\ K_{n+1}(\omega_1,0) \\ . \\ . \\ K_{n+1}(\omega_{NT-1},0) \end{bmatrix} = \qquad (2.28)$$

$$\begin{pmatrix} \exp(-\varphi(\omega_0,0)(1-K_{0,n}^2) & \exp(-\varphi(\omega_0,\Delta\tau)(1-K_{1,n}^2) & \ldots & \exp(-\varphi(\omega_0,(NT-1)\Delta\tau)(1-K_{n,n}^2) \\ \exp(-\varphi(\omega_1,0)(1-K_{0,n}^2) & \exp(-\varphi(\omega_1,\Delta\tau)(1-K_{1,n}^2) & \ldots & \exp(-\varphi(\omega_1,(NT-1)\Delta\tau)(1-K_{n,n}^2) \\ . & . & \ldots & . \\ . & . & \ldots & . \\ . & . & \ldots & . \\ \exp(-\varphi(\omega_{NT-1},0)(1-K_{0,n}^2) & \exp(-\varphi(\omega_{NT-1},\Delta\tau)(1-K_{1,n}^2) & \ldots & \exp(-\varphi(\omega_{NT-1},(NT-1)\Delta\tau)(1-K_{n,n}^2) \end{pmatrix} \begin{pmatrix} r_{n,0} \\ r_{n,1} \\ \vdots \\ r_{n,NT-1} \end{pmatrix}$$

For a given iteration number n, we can solve for the corresponding reflectivity series by solving Eq. (2.28) employing standard least squares inversion. Note that $K_0^2=0$. After a new estimate of the reflectivity series has been obtained, an update of $K_{i,n}{}^2$ can be obtained by solving the forward problem in Eq. (2.18). Iterations are carried out until the relative change in reflectivity is below a certain user threshold.

3.Regularisation and stabilisation

Regularisation considers the properties of the mapping operator M from the mathematical viewpoint: Whether the numerical instability comes from the singularity, and whether the singular operator can be modified to stabilize the computation. To better understand the concept, we briefly repeat Eq. (1.2).

$\vec{K}$ is a (Nx1) vector, M is a (NxN) matrix and $\vec{r}$ is a (Nx1) vector. Let $\vec{K}$ be the desired seismic output data while the actual output from Eq. (2.28) is $\vec{S} = M\,\vec{r}$. We want to compute a reflectivity per depth unit series $\vec{r}$ such that the difference $\vec{\Sigma}$ between the actual output $\vec{S}$ and the predicted seismic output data $\vec{K}$ is minimum in the least square sense. Therefore, the error $\vec{\Sigma}$ with respect to parameter vector $\vec{r}$ is $\vec{\Sigma} = \vec{K} - \vec{S} = \vec{K} - M\vec{r}$. And the cumulative squared error:

$$\vec{\Sigma}^T\,\vec{\Sigma} = (\vec{K} - M\vec{r})^T * (\vec{K} - M\vec{r}) =$$

$$(\vec{K}^T * \vec{K} - \vec{K}^T M\vec{r}) \; - r^T M^T K^T + (\vec{r}^T M^T * M\vec{r}) \qquad (3.1)$$

Where T denotes matrix transpose and * denotes complex conjugate.

We want to estimate a reflectivity per depth unit series $\vec{r}$ such that the quantity $\vec{\Sigma}^T\,\vec{\Sigma}$ is minimum. This condition leads to setting the derivative of $\vec{\Sigma}^T\,\vec{\Sigma}$ with respect to $\vec{r}$ to zero. Differentiate both sides of eq. (3.2) with respect to $\vec{r}$ and observe the requirement for least square procedure minimization that

$$\frac{\delta \vec{\Sigma}^{T} \vec{\Sigma}}{\partial \vec{r}} = -\vec{K}^{T} * M + r^{T} * M^{T} * M = 0 \qquad (3.2)$$

Because $\vec{r}^{T*}$ is complex valued, $\frac{\delta \vec{r}^{T*}}{\delta \vec{r}} = 0$. Thus applying matrix transpose and rearranging the terms of eq. (3.1)

$$\begin{aligned} &(M^{T} * M)^{T} * \vec{r} = M^{T}\vec{K} \Rightarrow (M^{T}M)\vec{r} = M^{T} * \vec{K} \\ &\Rightarrow \vec{r} = (M^{T} * M)^{-1} M^{T}\vec{K} \end{aligned} \qquad (3.3)$$

Eq (3.3) will give us the reflectivity per depth unit and from this we can calculate the impedance.

Damping constant when calculating reflectivity

To further understand the inversion we need to discuss how reflectivity per depth unit is computed and the introduction of the matrix M defined in Eq.(3.1). However an important aspect must be discussed first. The singularity of the matrix M^T*M makes it necessary to introduce a damping constant λ when calculating r. This λ is chosen out from the 'singular value decomposition' (svd) of the matrix M^T*M. Now we get an invertible new matrix:

$L=svd(M^T*M)$ giving $M^T*M + \lambda I$ (3.4)

I is a unitary matrix of the same order as the matrix M^T*M

$$\begin{aligned} &(M^{T} * M)^{T} * \vec{r} = M^{T}\vec{K} \Rightarrow (M^{T}M)\vec{r} = M^{T} * \vec{K} \\ &\Rightarrow \vec{r} = (M^{T} * M)^{-1} M^{T}\vec{K} \end{aligned}$$

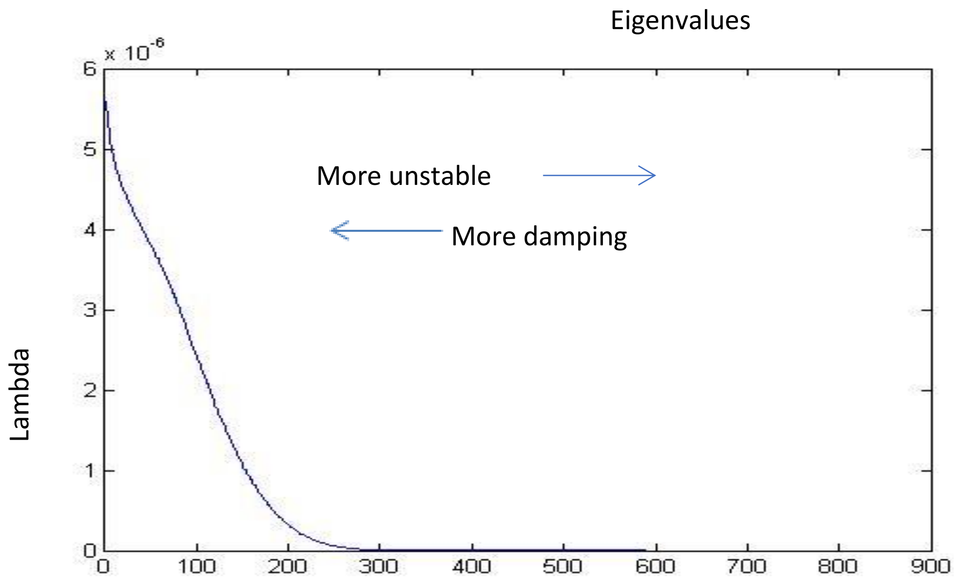

Fig.1 .Damping constant lambda as a function of the eigenvalues of M^T*M

We will choose lambda around (NT/2) eigenvalue of the matrix M^T*M to be able to use it in the inversion. The output reflectivity (r) will then depend on the value of lambda and introduce damping.

Fig.1 shows that when we choose smaller eigenvalues lambda will increase and r is more damped. When lambda increase we found that the effect of the inversion was less and over a threshold value had no effect at all. When we increase eigenvalues, lambda decrease. Then we get less damping but r is also more unstable, and can introduce noise.

It should be taken care to choose the right damping constant (λ) in order to perform the inversion. The choice must be related to noise level, choice of Butterworth filtering and scaling until one gets a satisfying result. We have not discussed this here, but plan to do it in future research.

Proceeding to the calculation we repeat that the procedure of inversion is formulated on the basis that the seismogram at the surface K(0,w) is a known quantity, while the reflectivity per depth unit series r(t) is the unknown which is to be determined. This seismogram k(w,0) can be field or synthetic seismic data.

In forward Q-modeling the reflectivity per depth unit series r(t) is known from the input layered model parameters. We can put up a model of parameters:

Layer 1	*Q* *200*	*Z (Depth)* *600*	*Density ρ* *1.9*	*Velocity* *4500*
Layer 2	*50*	*200*	*2.2*	*5000*
Layer 3	*200*	*400*	*2.4*	*3200*
Layer 4	*50*	*200*	*2.3*	*5000*
Layer 5	*200*	*500*	*2.3*	*4500*

Table 1. Five-layer model

In this approach we will use the same Q-values to calculate the synthetic seismogram K and the matrix M used to perform the inversion to get the reflectivity per depth unit series r. To avoid aliasing lower frequencies are used during the inversion process. Due to the singularity of the generated matrix M^T*M , a suitable damping constant λ is introduced in calculating the matrix $(M^T*M)^{-1}$. This suitable damping constant λ is chosen from the singular value decomposition as explained in eq (3.4).

For our discussion of the inversion we need to introduce the impedance. The relation between r and the impedance makes it possible to compute the impedance for every solution of r. If the acoustic impedance I_0 is known at z = o, (or at any depth), I is also uniquely determined as a function of τ. From Eq. (2.8) we can deduce after converting to two-way time:

$$I = I_0 \exp(2\int_0^{\tau} r(\tau) d\tau) \qquad (3.6)$$

where $I_0=\rho_0 v_0$ and $I= \rho v$. I/I_0 will give us a dimensionless relative impedance.

To repeat from Eq. (3.4) we can define a damping function taking the single value analysis where damp= λ I. Then we can define r as:

$$\vec{r} \rightleftarrows= ((M^T*\mathrm{M})+Damp\backslash(M^T)) * K$$

4.Seismic Wavelet estimation and regularisation

When we do inversion we need to know the seismic wavelet that has been used in the input data. The seismic wavelet combines source signature, receiver response, recording instrument effect and wave distortion within the surface media. We can express our seismic trace d as a convolutional model, the seismic trace r and the seismic wavelet w:

$$d(t)=w(t)*r(t)$$

If we have a well-log information, we should be able to estimate a wavelet by correlation between a seismic trace at the vicinity of the well location and a reflectivity series calculated, based on well-logs.

However, since we have no well log for our model, we must estimate the wavelet and the reflectivity series from our single equation. In practice we should make a reliable wavelet estimate first, independently from the reflectivity inversion, before we retrieve the reflectivity from the seismic trace. So we use a formula for a Ricker wavelet w(t) and convolve this with r(t):

$$w(t)=(2/(\sqrt{\pi}))\left(\frac{f2}{f3_c}\right)\exp\left(-\frac{f}{fc}\right)^2 \qquad (4.1)$$

This is done in the left plot on fig.2.depicting reflection forward seismogram after 5 iterations Eq.(2.18).Then from left we have 1.iteration and then 5.iteration in inversion. (Eq.(2.18)). Then we have the impedance (Eq.(3.6)). We have used different values for lambda obtaining several damping functions in inversion. We used the same Rickerpulse f_c =40 both for the modeling data and for the inversion.

```
Damp(i,j) = (1e-19)*(5.);
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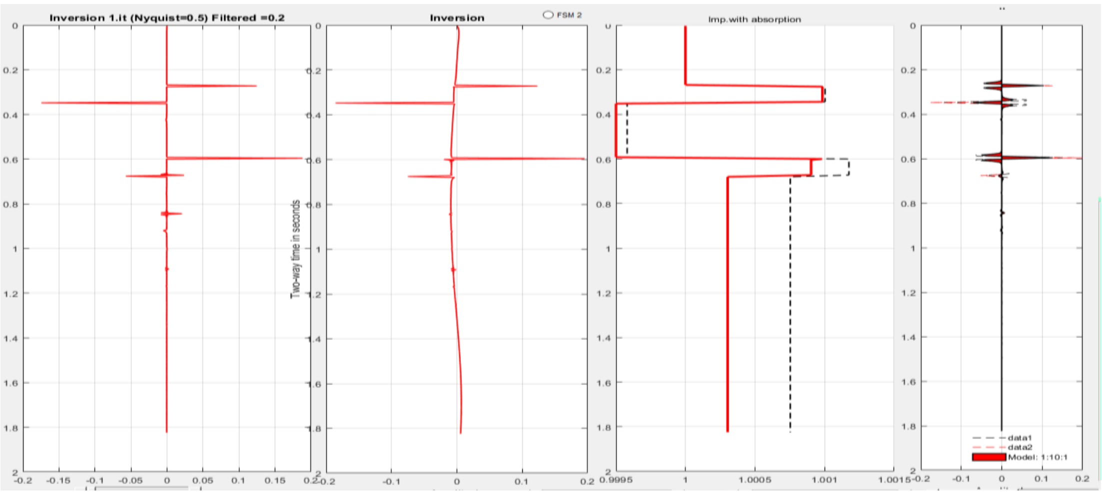

Fig.2 a.Left inversion 1. It, inversion 5 iterations, impedance, synthetics

```
Damp(i,j) = (1e-19)*(4.8);
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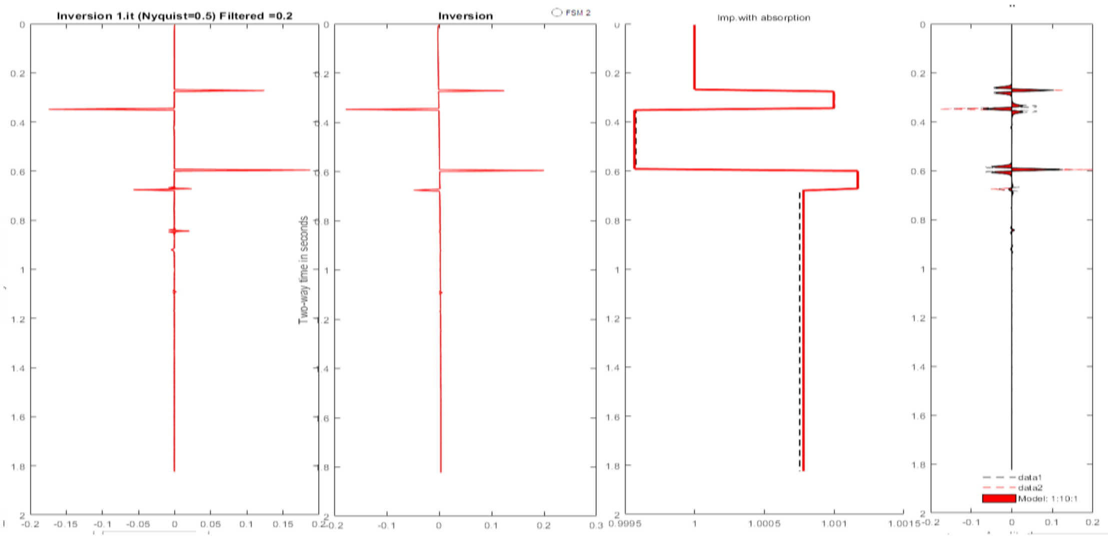

Fig.2 b.Left inversion 1. It, inversion 5 iterations, impedance, synthetics

```
Damp(i,j) = (1e-19)*(5.2);
```

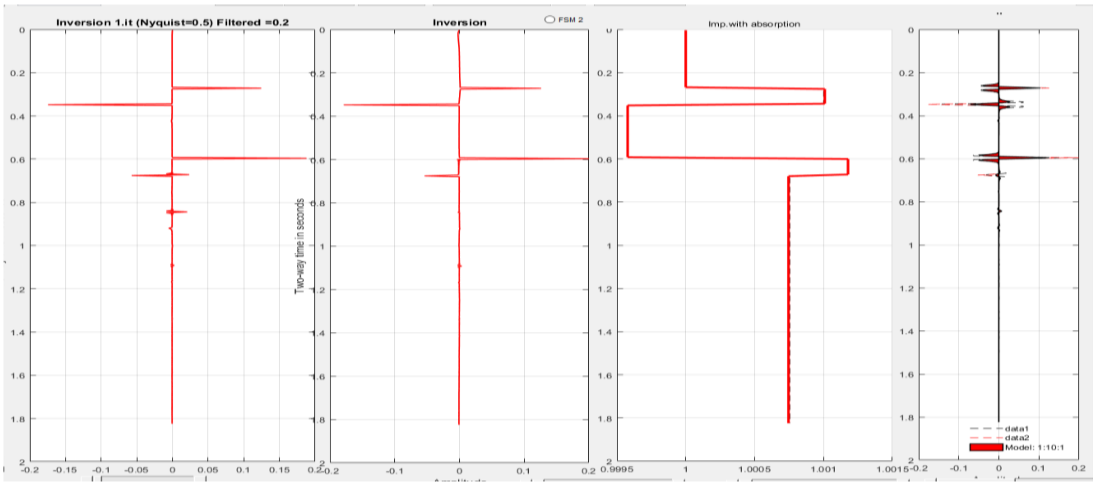

Fig.2 c.Left inversion 1. It, inversion 5 iterations, impedance, synthetics

Fig.2 gives r for different choices of the damping function. The damping function is calculated for eigenvalues chosen around NT/2. Fig.2.a. is for NT/2 giving damping 5. Fig.2b is for NT damping 4.8 and finally fig.2.c. NT and damping 5.2.
The impedance shows that the inversion went very well for eigenvalues a little bit different from NT/2.

Trying values for damping more different than for fig.2, we introduced higher values in fig.3. For lambda we used NT/2 as eigenvalue and added and subtracted 1 from the result.

```
Damp(i,j) = (1e-19)*(lambda+1);
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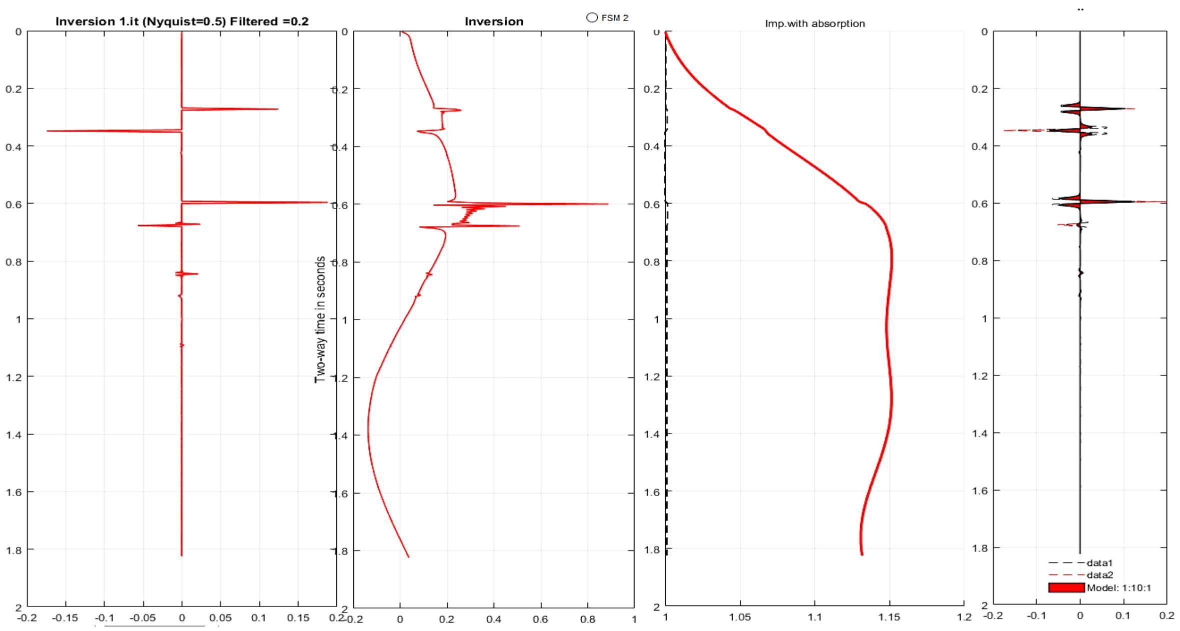

Fig.3 a.Left inversion 1. It, inversion 5 iterations, impedance, synthetics

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Damp(i,j) = (1e-19)*(lambda-1);
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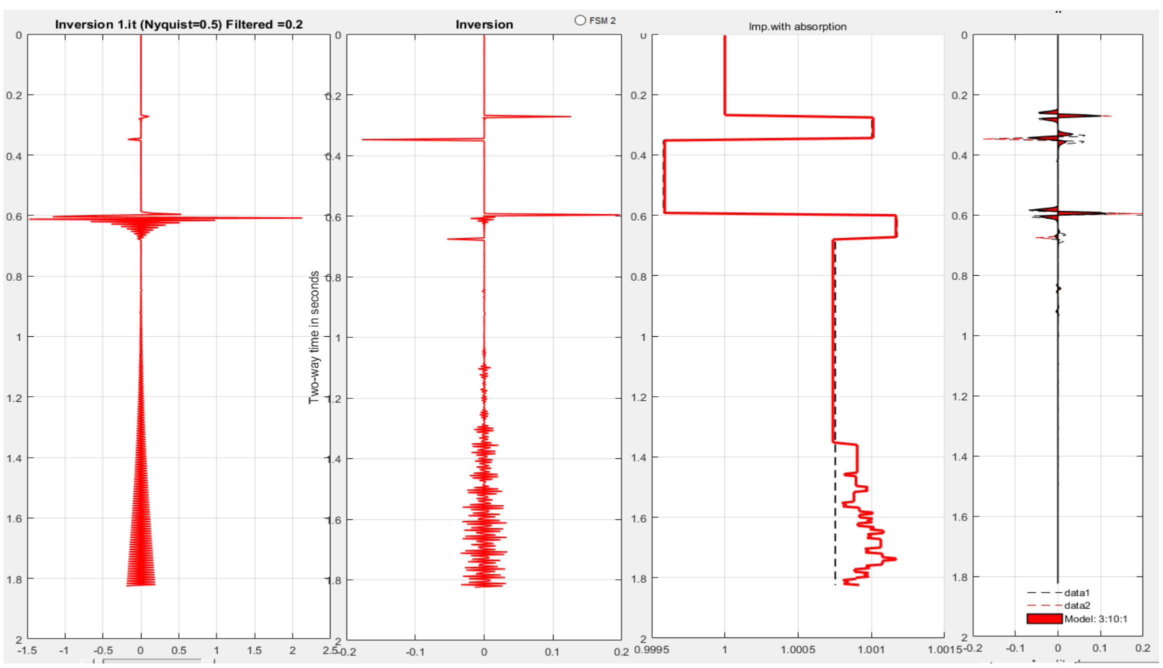

Fig.3 b.Left inversion 1. It, inversion 5 iterations, impedance, synthetics

5.Free surface – water bottom multiples

In the outline above we have considered the source and the receiver in the seismic model to be at the surface of the water . If we consider that the source and receiver are deeper in the water layer. Then we have to take into account surface-related multiples. We could implement water-bottom multiples due to air-water reflection.

If we assume that t_w represents two-way vertical travel time in the water layer, the total field $P_l = P(w_l)$ recorded at the receiver including multiples can then be written as:

$$P_l = K_{l,j=0}\,(1 - r\exp(iw_l t_w) + r\exp(iw_l t_w) + \ldots = \frac{K_{l,j=0}}{1 + r\exp(-iw_l t_w)} \qquad (5.1)$$

Eq (5,1) will introduce water bottom multiples in K simply by multiplying K with P in the forward computations. To remove multiples in the inversion we simply multiply the inverted K with the inverse of P.

```
Damp(i,j) = (1e-19)*4.8;
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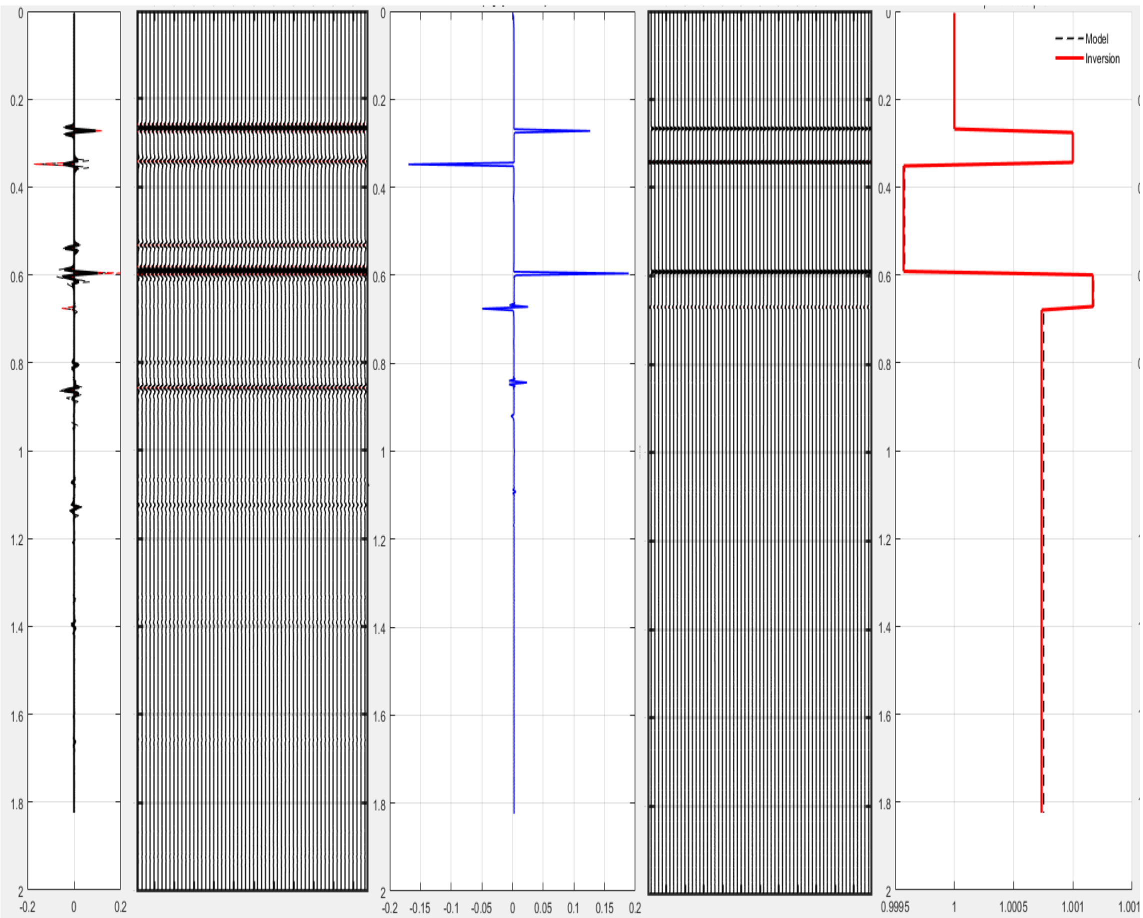


Fig.4.a.Synthetics, Wiggles plot:Synthetics with water bottom multiples lambda=NT/2 (Damping is 4.8). The inversion r(t) after 5 iterations. Then wiggles plot of inversion after 5 iterations. Impedance to right.

We both have a sharpening and removing of all multiples in the inversion. The impedance show that we got exactly the original. Fig.4.b give the same computations with damping 4.8. We got a better inversion.

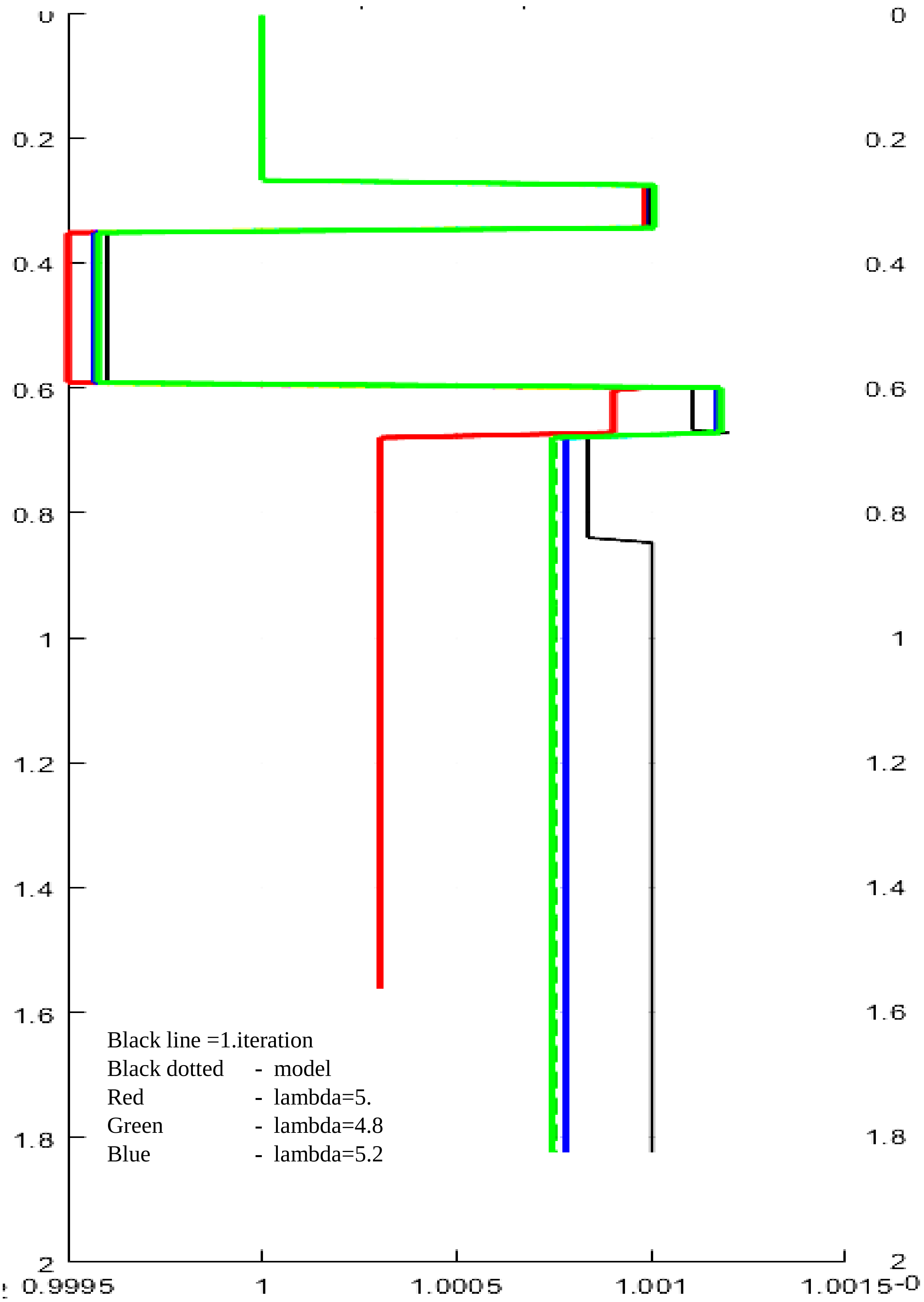
6.Impedance for lambda and iterations

In the discussion above we found that lambda=4.8 gave an inversion indistinguishable from the model. In fig.5 we have graphs of the impedance inversion for different choices of lambda and can see that values both more than lambda=5 and less gave a pretty good inversion. Lambda=5 is the solution for eigenvalue=NT/2. We can then conclude that the eigenvalues of the svd for the model gave convergence around NT/2. The black graph is the first iteration in the inversion So we then can suspect that we would get different solution for the inversion dependent on how many iterations we use. This is also an aspect of the Riccati inversion that was discussed by Gjevik (1976) and Nielsen and Gjevik e (1978). In this paper I will not discuss the aspect of iterations for the model. I have considered the inversion for 1th. And. 5th. iteration only.

7.Conclusion

The success of non-linear seismic inversion depends on the choice of an eigenvalue giving a certain damping function in the inversion procedure. As long as we know the shot-pulse and the depth of the transducer we can easily remove other effects.

Fig.5. Impedance inversion for different lambda. Black line is 1.iteration. All other graphs are with 5 iterations. Green graph (lambda=4.8) is indistinguishable from the model. (dotted line partly visible.)



Acknowledgement

It is a pleasure for me to express my gratitude to professor emeritus Bjørn Gjevik who first introduced me to the inversion with the Riccati equation and professor Leiv Gelius at the University of Oslo who first introduced me to seismic least square inversion. I will thank them for help and advice all the way through the application of this modification of his original theory.

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